AEES Distance Learning Programme

Class IX Mathematics

Chapter 12 Herons Formula

Module 2 / 2

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In this section, we will find the areas of quadrilaterals and some plane figures using Heron's Formula.

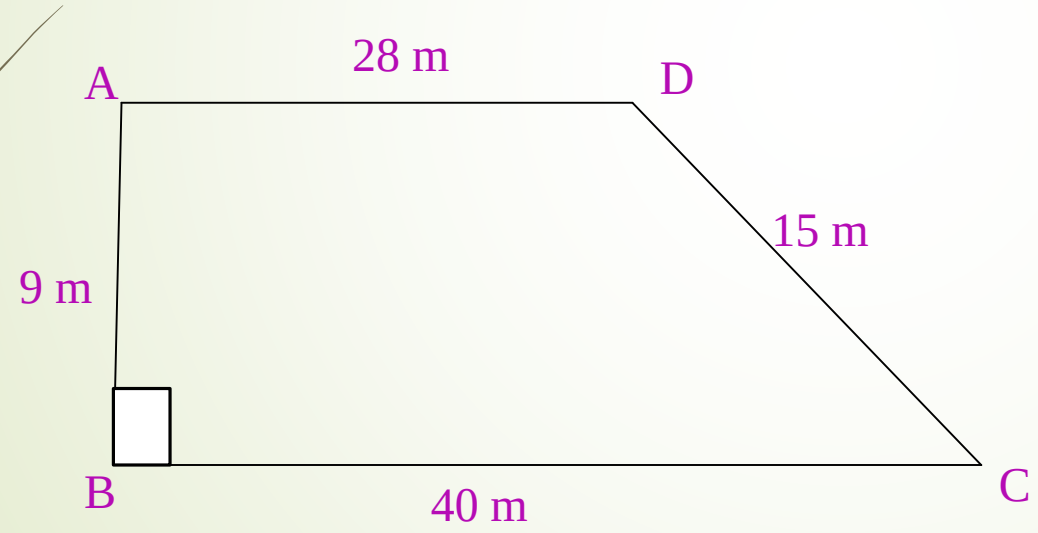
Suppose that a farmer has a land to be cultivated and he employs some laborers for this purpose on the terms of wages calculated by area cultivated per square meter.

How will he do this?

If a farmer wishes to find the area of his field which is in the shape of a quadrilateral. He needs to divide the quadrilateral in triangular parts and then use the formula for area of the triangle.

We will concentrate on such type of problems in the following examples.

Ex. 1 Students of a school staged a rally for cleanliness campaign. They walked through the lanes in two groups , One group walked through the lanes AB, BC and CA while other through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If AB = 9m BC = 40 m and CD = 15m, DA = 28 m and B= 90. Which group cleaned more area and by how much? Find the total area cleaned by the students?



In Δ ABC

∠B = 90 (By Pythagoras Theorem)

$AC^2 = AB^2 + BC^2$

$AC^2 = 9^2 + 40^2 = 1681$

AC=41m

ar(Δ ABC) = ½ x b x h

= ½ x 40 x 9

= 180 m^2

Now in Δ ABC, AC= 41m AD=28m DC= 15m

$S = \frac{a+b+c}{2} = \frac{28+41+15}{2} = 42m$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{42(42-28)(42-41)(42-15)}$

$A = \sqrt{42 * 14 * 1 * 27}$

A = 126 m^2

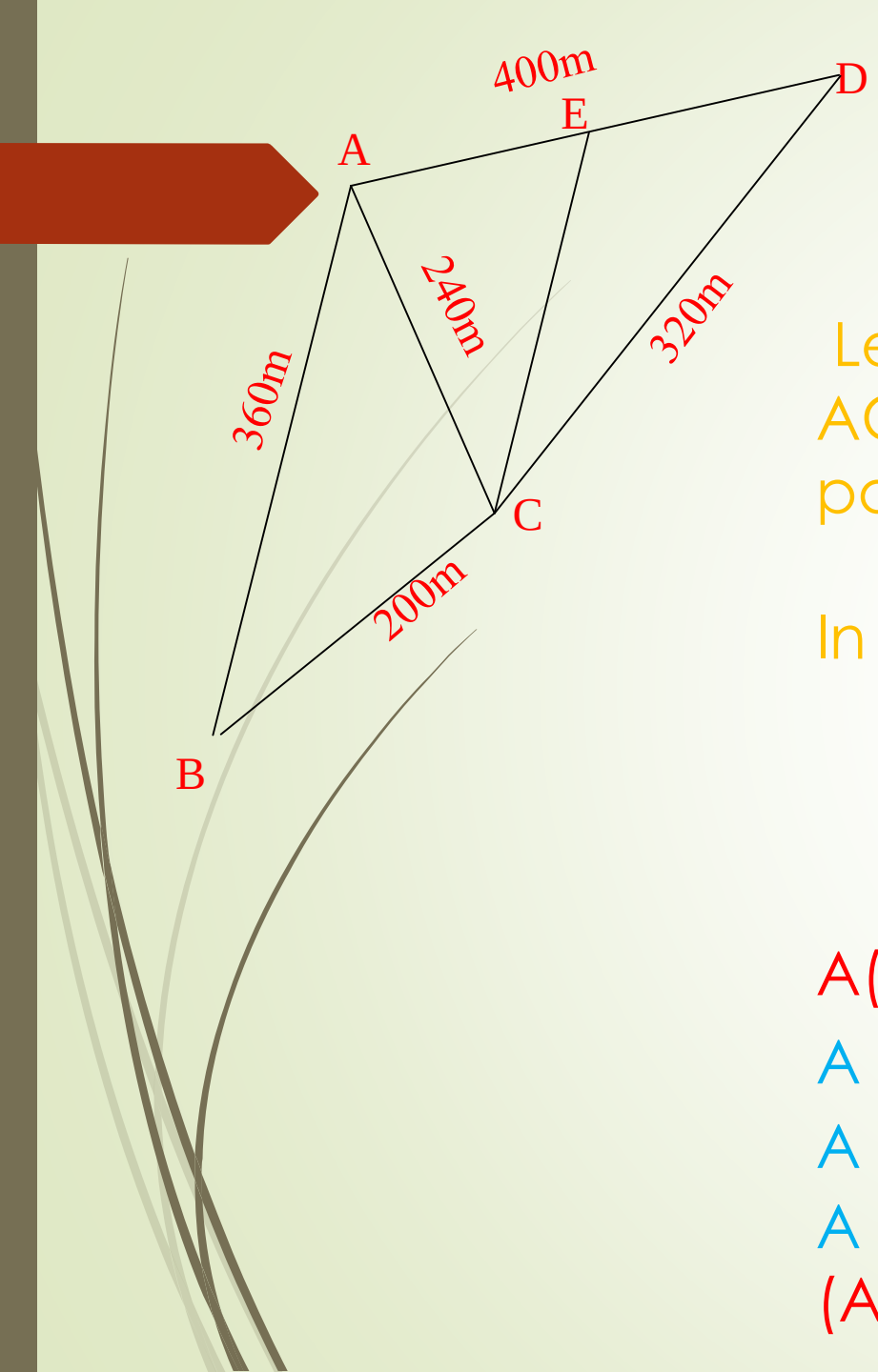
So,first group cleaned $80m^2$ and second group cleaned $126m^2$

Total area = (180+126) m^2

= $306m^2$

First group cleaned 180-126 = $56m^2$ more than the second group.

Ex 2. Kamla has a triangular field with sides 240 m, 200 m and 360 m where she grew wheat. In another triangular field with sides 240 m , 320m. 400m adjacent to the previous field she wanted to grow potatoes and onions. She divided the field in two parts by joining the midpoint of the longest side to the opposite vertex and grew potatoes in one part and onion in the other part. How much area (in hectares)has been used for wheat , potatoes and onions?



Let Δ ABC be the field where wheat is grown and Δ ACD be the field which has been divided into two parts by joining C to the midpoint of AD

In Δ ABD a= 200m

b= 240m

c=360 m

$S = \frac{a+b+c}{2} = \frac{200+240+360}{2} = 400\text{m}$

$A(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$A = \sqrt{400(400-200)(400-240)(400-360)}$

$A = \sqrt{400 * 200 * 160 * 40}$

$A = 1600\sqrt{2}\ m^2 = 1.6\sqrt{2}$ hectares = 2.26 hectares (Approx.)

Now For Δ ACD

$$S = \frac{a+b+c}{2} = \frac{240+400+320}{2} = 480\text{m}$$

$$\text{Area}(\Delta ACD) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{480(480-240)(480-400)(480-320)}$$

$$A = \sqrt{480 * 240 * 80 * 160}$$

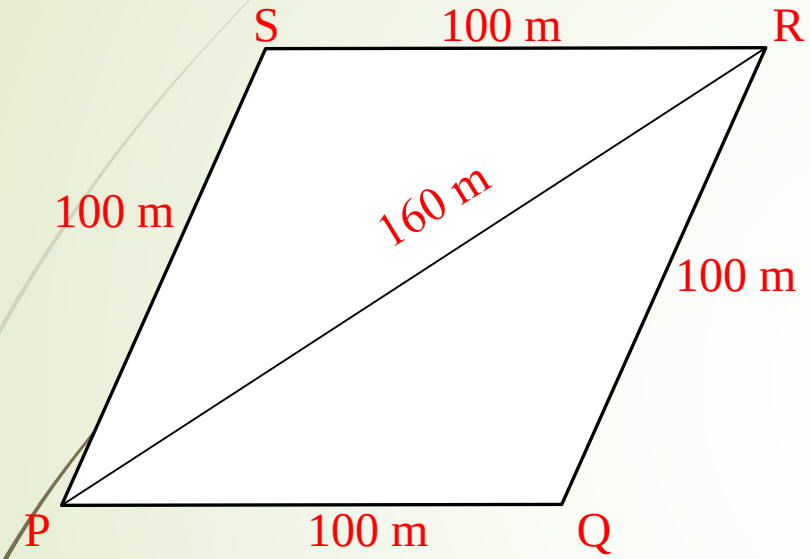
$$A = 38400\ \text{m}^2 = 3.84\ \text{hectares}$$

Area of (Δ AEC) = Area of (Δ DEC) (same base and same height)

:. Area for growing potatoes = area for growing onions

3.84/2 =1.92 hectares

Ex 3. Sanya has a piece of land which is in the shape of a rhombus. She wants her one daughter and one son to work on the land and produce different crops, she divided the land in two equal parts. If the perimeter of the land is 400m and one of the diagonals is 160 m . How much area each of them will get for their farms?



Perimeter of rhombus = 4 x S

$\Rightarrow$ 400 = 4 x S

$\Rightarrow$ S= 400/4 =100m

Each side = 100m

Let PR = 160m (diagonal)

In Δ PQR,

$$S = \frac{a+b+c}{2} = \frac{160+100+100}{2} = 180m$$

$$A(\Delta PQR) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$A = \sqrt{180(180-100)(180-100)(180-160)}$$

$$A = \sqrt{180 * 80 * 80 * 20}$$

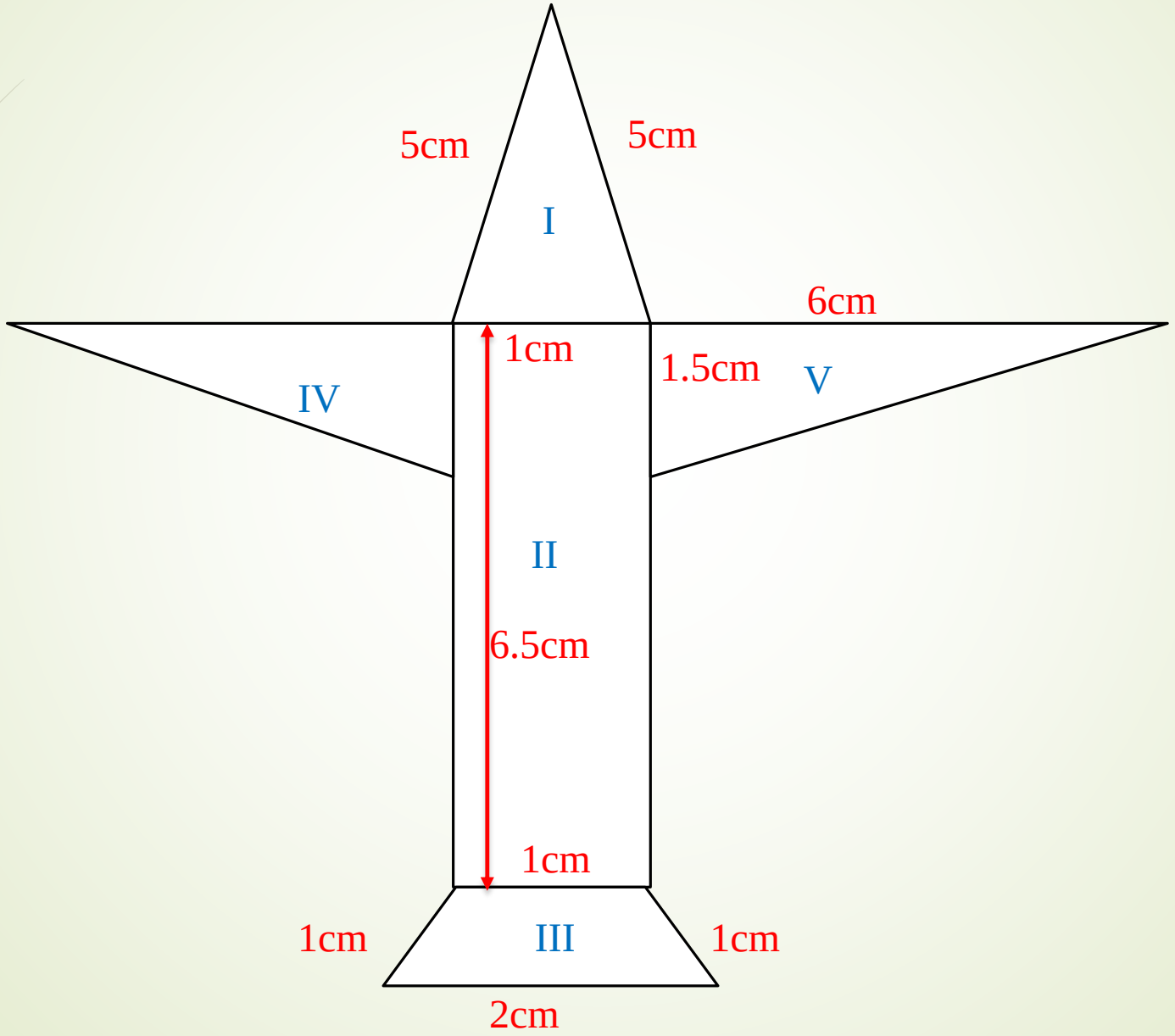
$$A = 4800\ m^2$$

But ar(Δ PQR) = ar(Δ PSR)

ar(Δ PSR)= 4800 m^2

Thus, each of them will get an area of 4800 m^2

Ex 4. Radha made a picture of an airplane with colored paper as shown in Figure. Find the total area of the paper used.



Area of I

$$S = \frac{a+b+c}{2} = \frac{5+5+1}{2} = \frac{11}{2}\text{cm}$$

Area of part I = $\sqrt{s(s-a)(s-b)(s-c)}$

$$A = \sqrt{\frac{11}{2}\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-1\right)}$$

$$A = \sqrt{\frac{11}{2} * \frac{1}{2} * \frac{1}{2} * \frac{9}{2}}$$

$$A = \frac{3}{4}\sqrt{11}\ \text{cm}^2$$

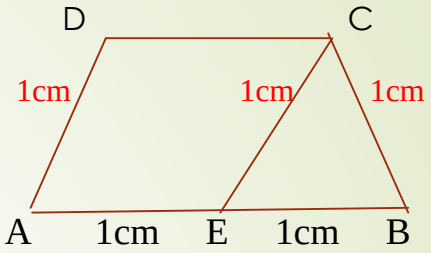
A = 2.482 (Approx.)

Area of II = L x B

= 6.5 cm x 1 cm

= 6.5 cm^2

Area of III= C



$ar(\Delta BEC) = \frac{\sqrt{3}}{4} X\ 1 = \frac{\sqrt{3}}{4}\ cm^2$

But $ar(\Delta BEC) = \frac{1}{2} X\ b\ X\ h = \frac{\sqrt{3}}{4}$

$h = \frac{\sqrt{3}}{4} \times 2$

$h = \frac{\sqrt{3}}{2}\ cm^2$

Ar(trapeziumABCD) =

$\frac{1}{2}\ (Sum\ of\ parallel\ sides\ )\ X\ height$

$= \frac{1}{2}\ X\ (1 + 2)\ X \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{4}\ cm^2$

Area of III $= \frac{3\ X\ 1.73}{4} = 1.3\ cm\ 2\ (Approx.)$

Area of IV $= \frac{1}{2} \; X \; b \; X \; h$

$= \frac{1}{2} \; X \; 1.5 \; X \; 6$

$= 4.5 \text{ cm}^2$

Area of V = area of IV = 4.5 cm^2

Total area of the paper used

= 2.482 +6.5 +1.3 +4.5 +4.5

$= 19.3 \text{ cm}^2$ (Approx.)

Thank You